Code No: 133BD **R16**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B. Tech II Year I Semester Examinations, September/October - 2023
MATHEMATICS - IV
(Common to CE, EEE, ME, ECE, CSE, EIE, IT, MCT, MMT, AE, MIE, MSNT)

Time: 3 Hours **Max. Marks: 75**

Note: i) Question paper consists of Part A, Part B.
ii) Part A is compulsory, which carries 25 marks. In Part A, answer all questions.
iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART – A

(25 Marks)

1.a) Define a regular function and give an example for a regular function. [2]
b) Define Harmonic and conjugate harmonic functions. [3]
c) State Cauchy's integral formula. [2]
d) Define a pole of order $'m'$ and give an example for it. [3]
e) State fixed points or invariant points of a bilinear transformation. [2]
f) Explain the evaluation of the integral $\int_{-\infty}^{\infty} f(x)dx$ using contor integration when poles of f(z) do not lie on real axis. [3]
g) Define a periodic function. [2]
h) State Fourier integral theorem. [3]
i) Is the partial differential equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ parabolic equation? If not what type of equation it is? [2]
j) State one dimensional heat equation. [3]

PART – B

(50 Marks)

2. State and prove the necessary and sufficient conditions for a function $w = f(z)$ to be analytic. [10]

OR

3. Define an analytic function and hence find the analytic function whose imaginary part is $e^{-x}(xsiny - ycosy)$. [10]

4. Evaluate $\oint_C \frac{e^z\, dz}{(z+1)^4}$ where C is the circle $|z| = 2$. [10]

OR

5. Expand $f(z) = \frac{2}{(2z+1)^3}$ about a) $z = 0$ *and* b) $z = 2$ as Laurant's series. [10]

6. Evaluate, by Cauchy's residue theorem $\int_{-\pi}^{\pi} \frac{d\theta}{5+4sin\theta}$. [10]

OR

7. Find the bilinear transformation which maps the points $1, i, -1$ in the z-plane on to the points $i, 0, -i$ in the w-plane. Hence find the invariant points of this transformation. [10]

8. Write a notes on the Fourier series of odd and even functions. Find the Fourier series expansion of the function $f(x) = |cos x|$ in the interval $[-\pi, \pi]$, $f(x+2\pi) = f(x) \forall x \in \mathbb{R}$. [10]

OR

9. Find the Fourier Transform of $f(x) = \begin{cases} 1 - x^2, |x| \le 1 \\ 0, \quad |x| > 1 \end{cases}$. Hence evaluate $\int_0^\infty \frac{x cos x - sin x}{x^3} cos\frac{x}{2} dx$. [10]

10. Solve the equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y} + 2u$ by method of separation of variables, subject to the conditions $u = 0$ and $\frac{\partial u}{\partial x} = 1 + e^{-3y}$, when $x=0$ for all values of y. [10]

OR

11. Solve the one-dimensional wave equation by the method of separation of variables and hence find all the possible solutions. [10]

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