

**R16**

Code No: 133BD

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech II Year I Semester Examinations, September/October - 2023

MATHEMATICS - IV

(Common to CE, EEE, ME, ECE, CSE, EIE, IT, MCT, MMT, AE, MIE, MSNT)

Time: 3 Hours

Max. Marks: 75

Note: i) Question paper consists of Part A, Part B.

ii) Part A is compulsory, which carries 25 marks. In Part A, answer all questions.

iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

**PART - A**

**(25 Marks)**

- 1.a) Define a regular function and give an example for a regular function. [2]
- b) Define Harmonic and conjugate harmonic functions. [3]
- c) State Cauchy's integral formula. [2]
- d) Define a pole of order 'n' and give an example for it. [3]
- e) State fixed points or invariant points of a bilinear transformation. [2]
- f) Explain the evaluation of the integral  $\int_{-\infty}^{\infty} f(x)dx$  using contour integration when poles of  $f(z)$  do not lie on real axis. [3]
- g) Define a periodic function. [2]
- h) State Fourier integral theorem. [3]
- i) Is the partial differential equation  $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$  parabolic equation? If not what type of equation it is? [2]
- j) State one dimensional heat equation. [3]

**PART - B**

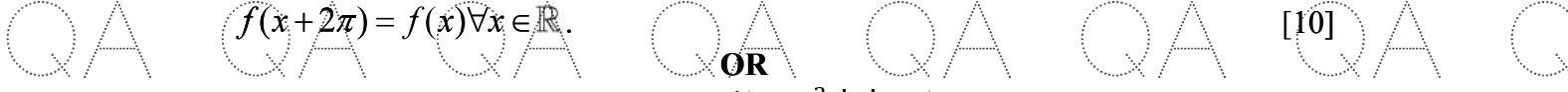
**(50 Marks)**

2. State and prove the necessary and sufficient conditions for a function  $w = f(z)$  to be analytic. [10]
- OR**
3. Define an analytic function and hence find the analytic function whose imaginary part is  $e^{-x}(x \sin y - y \cos y)$ . [10]
  4. Evaluate  $\oint_C \frac{e^z dz}{(z+1)^4}$  where C is the circle  $|z| = 2$ . [10]
- OR**
5. Expand  $f(z) = \frac{2}{(2z+1)^3}$  about a)  $z = 0$  and b)  $z = -2$  as Laurent's series. [10]
  6. Evaluate, by Cauchy's residue theorem  $\int_{-\pi}^{\pi} \frac{d\theta}{5+4\sin\theta}$ . [10]
- OR**
7. Find the bilinear transformation which maps the points  $1, i, -1$  in the  $z$ -plane on to the points  $i, 0, -i$  in the  $w$ -plane. Hence find the invariant points of this transformation. [10]



8. Write a notes on the Fourier series of odd and even functions. Find the Fourier series expansion of the function  $f(x) = |\cos x|$  in the interval  $[-\pi, \pi]$ ,

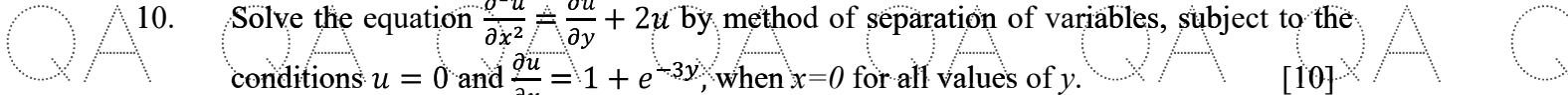
$f(x+2\pi) = f(x) \forall x \in \mathbb{R}$ . [10]



**OR**

9. Find the Fourier Transform of  $f(x) = \begin{cases} 1 - x^2, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$ . Hence evaluate

$\int_0^\infty \frac{x \cos x - \sin x}{x^3} \cos \frac{x}{2} dx$ . [10]



10. Solve the equation  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial y} + 2u$  by method of separation of variables, subject to the conditions  $u = 0$  and  $\frac{\partial u}{\partial x} = 1 + e^{-3y}$ , when  $x=0$  for all values of  $y$ . [10]

**OR**

11. Solve the one-dimensional wave equation by the method of separation of variables and hence find all the possible solutions. [10]

